3.6: Categorical Syllogisms

As we've said, Aristotelian Logic limits itself to evaluating arguments all of whose propositions—premises and conclusion—are categorical. There is a further restriction: Aristotelian Logic only evaluates categorical syllogisms. These are a special kind of argument, meeting the following conditions:

A categorical syllogism is a deductive argument consisting of three categorical propositions (two premises and a conclusion); collectively, these three propositions feature exactly three classes; each of the three classes occurs in exactly two of the propositions.

That's a mouthful, but an example will make it clear. Here is a (silly) categorical syllogism:

All chipmunks are Republicans.
Some Republicans are golfers.
Therefore, some chipmunks are golfers.

This argument meets the conditions in the definition: it has three propositions; there are exactly three classes involved (chipmunks, Republicans, and golfers); and each of the three classes occurs in exactly two of the propositions (check it and see).

There is some special terminology for the class terms and premises in categorical syllogisms. Each of the three class terms has a special designation. The so-called major term is the term that appears in predicate position in the conclusion; in our silly example, that's ‘golfers’. The minor term is the term that appears in subject position in the conclusion; in our example, that’s ‘chipmunks’. The middle term is the other one, the one that appears in each of the premises; in our example, it's 'Republicans'.

The premises have special designations as well. The major premise is the one that has the major term in it; in our
example, that’s ‘Some Republicans are golfers’. The **minor premise** is the other one, the one featuring the minor term; in our example, it’s ‘All chipmunks are Republicans’.

Final restriction: categorical syllogisms must be written in standard form. This means listing the premises in the correct order, with the major premise first and the minor premise second. If you look at our silly example, you’ll note that it’s not in standard form. To fix it, we need to reverse the order of the premises:

- Some Republicans are golfers.
- All chipmunks are Republicans.
- Therefore, Some chipmunks are golfers.

An old concern may arise again at this point: in restricting itself to such a limited class of arguments, doesn’t Aristotelian Logic run the risk of not being able to evaluate lots of real-life arguments that we care about? The response to this concern remains the same: while most (almost all) real-life arguments are not presented as standard form categorical syllogisms, a surprising number of them can be translated into that form. Arguments with more than two premises, for example, can be rewritten as chains of two-precise sub-arguments. As was the case when we raised this concern earlier, we will set aside the messy details of exactly how this is accomplished in particular cases.

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**Logical Form**

As we said at the outset of our exploration of deductive logic, there are three things such a logic must do: (1) tame natural language; (2) precisely define logical form; and (3) develop a way to test logical forms for validity. Until now, we’ve been concerned with the first step. It’s (finally) time to proceed to the second and third.

The logical form of a categorical syllogism is determined by two features of the argument: its **mood** and its **figure**. First, mood. The mood of a syllogism is determined by the types of categorical propositions contained in the argument, and the order in which they occur. To determine the mood, put the argument into standard form, and then simply list the types of categoricals (A, E, I, O) featured in the order they occur. Let’s do this with our silly example:

- Some Republicans are golfers.
- All chipmunks are Republicans.
- Therefore, some chipmunks are golfers.

From top to bottom, we have an I, an A, and an I. So the mood of our argument is IAI. It’s that easy. It turns out that there are 64 possible moods—64 ways of combining A, E, I, and O into unique three-letter combinations, from AAA to OOO and everything in between.

The other aspect of logical form is the argument’s figure. The figure of a categorical syllogism is determined by the arrangement of its terms. Given the restrictions of our definition, there are four different possibilities for standard form syllogisms. We will list them schematically, using these conventions: let ‘S’ stand for the minor term, ‘P’ stand for the major term, and ‘M’ stand for the middle term. Here are the four figures:

<table>
<thead>
<tr>
<th>(i)</th>
<th>MP</th>
<th>(ii)</th>
<th>PM</th>
<th>(iii)</th>
<th>MP</th>
<th>(iv)</th>
<th>PM</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM</td>
<td>MS</td>
<td>SM</td>
<td>MS</td>
<td>SP</td>
<td>MS</td>
<td>SP</td>
<td>MS</td>
</tr>
</tbody>
</table>
Again, the only thing that determines figure is the arrangement of terms—whether they appear in subject or predicate position in their premises. In our schemata, that the letter is listed first indicates that the term appears in subject position; that it appears second indicates that it’s in predicate position. So, in the first figure, in the major premise (the first one), the middle term (M) is in subject position and the major term (P) is in predicate position. Notice that for all four figures, the subject and predicate of the conclusion remains the same: this is because, by definition, the minor term (S) is the subject of the conclusion and the major term (P) its predicate.

Returning to our silly example, we can determine its figure:

Some Republicans are golfers.
All chipmunks are Republicans.
Therefore, some chipmunks are golfers.

Perhaps the easiest thing to do is focus on the middle term, the one that appears in each of the premises—in this case, ‘Republicans’. It occurs in subject position in the major premise, then predicate position in the minor premise. Scanning the four figures, I just look for the one that has ‘M’ listed in first position on the top, then second position in the middle. That’s the first figure.

So the mood of our sample argument is IAI, and it’s in the first figure. Logical form is just the mood and figure, and conventionally, we list logical forms like this: IAI-1 (the mood, a dash, then a number between 1 and 4 for the figure).

There are 4 figures and 64 moods. That gives us 256 possible logical forms. It turns out that only 15 of these are valid. We need a way to test them. It is to that task we now turn.

The Venn Diagram Test for Validity

To test syllogistic forms for validity, we proceed in three steps:

1. Draw three overlapping circles, like this:

   ![Venn Diagram]

   That gives us one circle for each of the three terms in the syllogism: minor (S), major (P), and middle (M).

2. Depict the assertions made by the premises of the syllogism on this diagram, using shading and Xs as
appropriate, depicting the individual A, E, I, or O propositions in the usual way:

![Diagram of A and E propositions]

Each of the premises will be a proposition concerning only two of the three classes (S, P, and M). The major premise will concern M and P (in some order); the minor premise will concern M and S (in some order). How the circles will be labeled (with S, M, P) will depend on these particulars.

3. After the premises have been depicted on the three-circle diagram, we look at the finished product and ask, “Does this picture entail the truth of the conclusion?” If it does, the form is valid; if it does not, it is invalid.

In the course of running the test, we will keep two things in mind—one rule of thumb and one convention:

Rule of Thumb: In step 2, depict universal (A and E) premises before particular (I and O) ones (if there’s a choice).

Convention: In cases of indeterminacy, draw Xs straddling boundary lines.

We need to explain what “indeterminacy” amounts to; we will in a moment. For now, to make all this more clear, we should run through some examples.

Let’s start at the beginning (alpha-numerically): AAA-1. We want to test this syllogistic form for validity. What does an argument of this form look like, schematically? Well, all three of its propositions are universal affirmatives, so they’re all of the form All __ are __. We have:

All __ are __
All __ are __
Therefore, all __ are __

That’s what the mood (AAA) tells us. We have to figure out how to fill in the blanks with S, P, and M. The figure tells us how to do that. AAA-1: so, first figure. That looks like this:
(i)  \[
\begin{align*}
  &\text{MP} \\
  &\text{SM} \\
  &\text{SP}
\end{align*}
\]

So AAA-1 can be schematically rendered thus:

All M are P.
All S are M.
Therefore, all S are P.

To test this form for validity, we start with step 1, and draw three circles:

In step 2, we depict the premises on this diagram. (We’re supposed to keep in mind the rule of thumb that, given a choice, we should depict universal premises before particular ones, but since both of the premises are universals, this rule does not apply to this case.) We can start with the major premise: All M are P. On a regular two-circle Venn diagram, that would look like this:

The trick is to transfer this two-circle diagram onto the three-circle one. In doing so, we keep in mind that all the parts of M that are outside of P must be shaded. That gives us this:
Note that in the course of shading out the necessary regions of M, we shaded out part of S. That's OK. Those members of the S class are Ms that aren't Ps; there's no such thing, so they have to go.

Next, we depict the minor premise: All S are M. With two circles, that would look like this:

Transferring that onto the three circle diagram means shading all the parts of S outside of M:

Step 2 is complete: we have depicted the assertions made by the premises. In step 3 we ask whether this diagram guarantees the truth of the conclusion. Well, our conclusion is All S are P. In a two-circle diagram, that looks like this:
Does our three-circle diagram guarantee the truth of All S are P? Focusing on the S and P circles, and comparing the two diagrams, there's a bit of a difference: part of the area of overlap between S and P is shaded out in our three-circle diagram, but it isn't in the two-circle depiction. But that doesn't affect our judgment about whether the diagram guarantees All S are P. Remember, this can be thought of as a claim that a certain kind of thing doesn't exist—an S that's outside the P circle. If there are any Ss (and there may not be), they will also be Ps. Our three-circle diagram does in fact guarantee this. There can't be an S that's not a P; those areas are shaded out. Any S you find will also be a P; it'll be in that little region in the center where all three circles overlap.

So, since the answer to our question is “yes”, the syllogistic form AAA-1 is valid. Trivial fact: all the valid syllogistic forms were given mnemonic nicknames in the Middle Ages to help students remember them. AAA-1 is called “Barbara”. No really. All the letters in the name had some meaning: the vowels indicate the mood (AAA); the other letters stand for features of the form that go beyond our brief investigation into Aristotelian Logic.

We should reflect for a moment on why this method works. We draw a picture that depicts the assertions made by the premises of the argument. Then we ask whether that picture guarantees the conclusion. This should sound familiar. We're testing for validity, and by definition, an argument is valid just in case its premises guarantee its conclusion; that is, IF the premises are true, then the conclusion must also be true. Our method mirrors the definition. When we depict the premises on the three-circle diagram, we're drawing a picture of what it looks like for the premises to be true. Then we ask, about this picture—which shows a world in which the premises are true—whether it forces us to accept the conclusion—whether it depicts a world in which the conclusion must be true. If it does, the argument is valid; if it doesn’t, then it isn’t. The method follows directly from the definition of validity.

To further illustrate the method, we should do some more examples. All-3 is a useful one. The mood tells us it's going to look like this:

All __ are __
Some __ are __
Therefore, some __ are __

And we're in the third figure:

(iii)  
MP
MS
SP
So we fill in the blanks to get the schematic form:

All M are P
Some M are S
Therefore, some S are P

We start the test of this form with the blank three-circle diagram:

Step 2: depict the premises. And here, our rule of thumb applies: depict universals before particulars. The major premise is a universal (A) proposition; the minor premise is a particular (I). So we depict the major premise first. That’s All M are P. We did this already. Recall that Barbara has the same major premise. So depicting that on the diagram gives us this:

Next, the minor premise: Some M are S. Recall, with particular propositions, we depict them using an X to indicate the thing said to exist. This proposition asserts that there is at least one thing that is both M and S:
We need to transfer this to the three-circle diagram. We need an X that is in both the M and S circles. If we look at the area of overlap between the two, we see that part of it has been shaded out as the result of depicting the major premise, so there’s only one place for the X to go:

![Three-circle diagram](https://human.libretexts.org/Bookshelves/Philosophy/Book%3A_Fundamental_Methods_of_Logic_(Knachel)/3%3A_Deductiv…)

Step 2 is complete: the premises are depicted. So we proceed to step 3 and ask, “Does this picture guarantee the conclusion?” The conclusion is Some S are P; that’s an assertion that there is at least one thing that is both S and P. Is there? Yes! That X that we drew in the course of depicting the minor premise is in the sweet spot—the area of overlap between S and P. It guarantees the conclusion. The argument is valid. (If you’re curious, its mnemonic nickname is ‘Datisi’. Weird, I know; it was the Middle Ages.)

That’s another successful use of the Venn diagram test for validity, but I want to go back a revisit some of it. I want us to reflect on why we have the rule of thumb to depict universal premises before particular ones. Remember, we had the universal major premise All M are P and the particular minor premise Some M are S. The rule of thumb had us depict them in that order. Why? What would have happened had we done things the other way around? We would have started with a blank three-circle diagram and had to depict Some M are S on it. That means an X in the area of overlap between M and S. That area, though, is divided into two sub-regions (labeled ‘a’ and ‘b’):
Where do I put my X—in region a or b? Notice, it makes a difference: if I put the X is region a, then it’s outside the P circle; if I put it in region b, then it’s inside the P circle. The question is: “Is this thing that the minor premise says exists a P or not a P?” I’m depicting a premise that only asserts Some M are S. That premise says nothing about P. It’s silent on our question; it gives us no guidance about how to choose between regions a and b. What to do? This is one of the cases of “indeterminacy” that we mentioned earlier when we introduced a convention to keep in mind when running the test for validity: In cases of indeterminacy, draw Xs straddling boundary lines. We don’t have any way of choosing between regions a and b, so when we draw our X, we split the difference:

This drawing indicates that there’s an X in there somewhere, either inside or outside the P circle, we don’t know which.

And now we see the reason for our rule of thumb—depict universals before particulars. Because if we proceed to depict the universal premise All M are P, we shade thus:
The shading erased half our X. That is, it resolved our question of whether or not the X should go in the P circle: it should. So now we have to go back and erase the half-an-X that’s left and re-draw the X in that center region and end up with the finished diagram we arrived at earlier:

We would’ve saved ourselves the trouble had we just followed the rule of thumb to begin with and depicted the universal before the particular—shading before the X. That’s the utility of the rule: sometimes it removes indeterminacy that would otherwise be present.

One more example to illustrate how this method works. Let’s test EOI-1. Noting that in the first figure the middle term is first subject and then predicate, we can quickly fill in the schema:

\[
\text{No M are P} \\
\text{Some S are not M.} \\
\text{Therefore, some S are P.}
\]

Following the rule of thumb, we depict the universal (E) premise first. No M are P asserts that there is nothing that is in both of those classes. The area of overlap between them is empty. With two circles, we have this:
Transferring this onto the three-circle diagram, we shade out all the area of overlap between the M and P circles (clipping off part of S along the way):

Next, the particular (O) premise: Some S are not M. This asserts the existence of something— namely, a thing that is an S but not an M. We need an X in the S circle that outside the M circle:

Moving to the three-circle diagram, though, things get messy. The area of S that’s outside of M is divided into two sub-regions (labeled ‘a’ and ‘b’):
We need an X somewhere in there, but do we put it in region a or region b? It makes a difference: if we put it in region b, it is a P; if we put it in region a, it is not. This is the same problem we faced before. We're depicting a premise—Some S are not M—that is silent on the question of whether or not the thing is a P. Indeterminacy. We can't decide between a and b, so we split the difference:

That X may be inside of P, or it may not; we don't know. This is a case in which we followed the rule of thumb, depicting the universal premise before the particular one, but it didn't have the benefit that it had when we tested All-3: it didn't remove indeterminacy. That can happen. The rule of thumb is in place because it sometimes removes indeterminacy; it doesn't always work, though.

So now that we’ve depicted the premises, we ask whether they guarantee the conclusion. Is the world depicted in our diagram one in which the conclusion must be true? The conclusion is Some S are P: it asserts that there is at least one thing that is both S and P. Does our picture have such a thing? There’s an X in the picture. Does it fit the bill? Is it both S and P? Well, uh... Maybe? That X may be inside of the area of overlap between S and P; then again, it may not be.

Oy. What do we say? It’s tempting to say this: we don’t know whether the argument is valid or not; it depends on where that X really is. But that’s not the correct response. Remember, we’re testing for validity—for whether or not the premises guarantee the conclusion. We can answer that question: they don’t. For a guarantee, we would need an X in
our picture that is *definitely* inside that middle region. We don’t have such an X. These premises leave open the possibility that the conclusion is true; they don’t rule it out. But that’s not enough for validity. For an argument to be valid, the premises must *necessitate* the conclusion, force it on us. These do not. The form EOI-1 is not valid. (Sad but true: the invalid syllogistic forms do not have mnemonic nicknames.)

### Exercises

1. Identify the logical form of the following arguments.

(a) Because some Wisconsinites are criminals and all criminals are scoundrels, it follows that some scoundrels are Wisconsinites.
(b) No surfers are priests, because all priests are men and some surfers are not men.
(c) Some authors are feminists, since some women are authors and some women are feminists.
(d) All mosquitoes are potential carriers of disease; therefore some mosquitoes are a menace to society, since all potential carriers of disease are a menace to society.
(e) Because some neo-Nazis are bloggers, some neo-Nazis are not geniuses, since no geniuses are bloggers.

2. Test the following syllogistic forms for validity.

(a) EAE-2
(b) EAE-3
(c) OAO-3
(d) EIO-4
(e) AOO-4
(f) IAI-1
(g) AII-1

3. Test the following arguments for validity.

(a) Some pirates are mercenaries; hence, some sailors are pirates, because all sailors are mercenaries.
(b) Some women are not nuns, but all nuns are sweethearts; it follows that some women are not sweethearts.
(c) Some Republicans are not politicians, for some Republicans are not Christians, and some Christians are not politicians.

4. Test the arguments in Exercise 1 for validity.